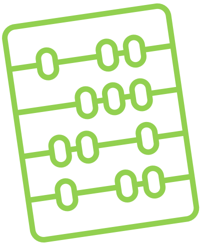
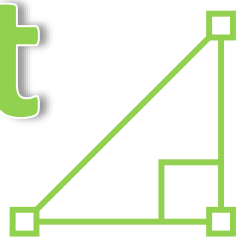


Pre-Calculus Must Knows!



Formula sheet

Pythagorean Identities

$$sin^2\theta + cos^2\theta = 1$$

$$1 + tan^2\theta = sec^2\theta$$

$$cot^2\theta + 1 = csc^2\theta$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Double-Angle Identities

$$\sin(2x) = 2sinxcosx$$

$$cos2x = cos^2x - sin^2x$$

$$= 2cos^2x - 1$$

$$= 1 - 2sin^2x$$

$$tan2x = \frac{sin2x}{cos2x}$$

$$tan2x = \frac{2sinxcosx}{cos^2x - sin^2x}$$

$$tan2x = \frac{2tanx}{1 - tan^2x}$$

Sum and Difference Identities

$$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$

$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$

$$\tan(A + B) = \frac{\tan(A) + \tan(B)}{1 - \tan(A)\tan(B)}$$

$$\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$$

Half-Angle Identities

$$sin\frac{x}{2} = \pm\sqrt{\frac{1 - cosx}{2}} \quad cos\frac{x}{2} = \pm\sqrt{\frac{1 + cosx}{2}} \quad tan\frac{x}{2} = \pm\sqrt{\frac{1 - cosx}{1 + cosx}}$$

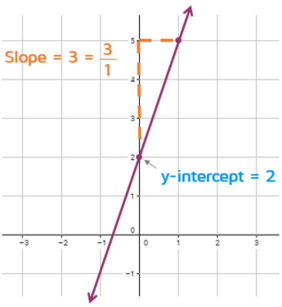
Linear Equations

An equation for a straight line

- Variables are only to the first power
- Variables are not multiplied together
- No variables in denominator

Slope-Intercept Form

$$y = mx + b$$



Where:

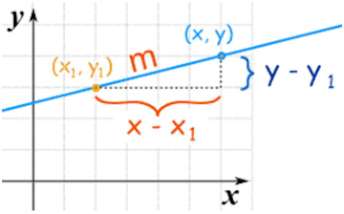
m = $\frac{rise\uparrow}{run\rightarrow}$ = slope

b = y-intercept

Point-Slope Form

$$y - y_1 = m(x - x_1)$$

Where:



General Form

$$Ax + By + C = 0$$

A and B cannot both be 0

★ Remember these are the same!!

$$y = 2x - 3$$

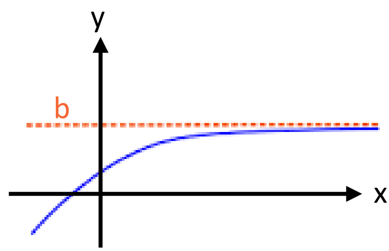
$$f(x) = 2x - 3$$

Asymptotes

An asymptote is a line that a curve approaches as it heads towards infinity.

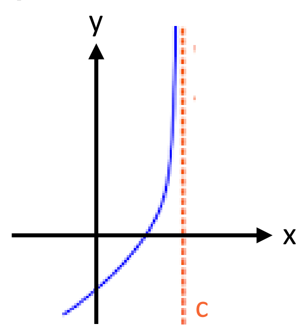
<u>Horizontal Asymptote</u>

As x goes to ∞ (or -∞) the curve approaches some constant value **b**



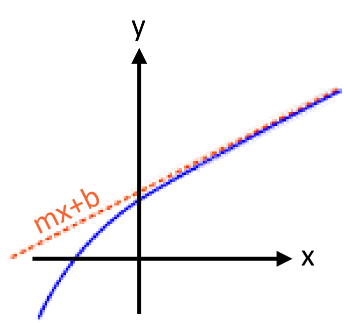
<u>Vertical Asymptote</u>

As x approaches some constant value **c** (from left or right) then the curve goes toward ∞ (or -∞)



<u>Slant/Oblique Asymptote</u>

As x goes to ∞ (or -∞) then the curve goes toward a line y=mx+b



★ Remember the important point is that the distance between the curve and asymptote **tends to zero**

Factoring

"Finding the factors"

aka. Finding what to multiply together to get an expression

Numbers have <u>factors</u>:

Factors

$$2 \times 3 = 6$$

Polynomials also have factors:

$$(x+3)(x+1) = x^2 + 4x + 3$$

Factor Factor

<u>Greatest Common Factor</u>

Ex. $8x^4 - 4x^3 + 10x^2$

$$2x^2(4x^2 - 2x + 5)$$

We can factor out a "2" and a "x^2" out of every term

Try this one:

$9x^2(2x+7) - 12x(2x+7)$

<u>Factoring By Grouping</u>

Ex. $x^5 + x - 2x^4 - 2$

$$(x^5 + x) - (2x^4 + 2)$$
$$x(x^4 + 1) - 2(x^4 + 1)$$
$$(x^4 + 1)(x - 2)$$

Try this one:

$x^5 - 3x^3 - 2x^2 + 6$

Special forms

$$\boldsymbol{a^2 + 2ab + b^2 = (a+b)^2}$$
$$\boldsymbol{a^2 - 2ab + b^2 = (a-b)^2}$$
$$\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$$
$$\boldsymbol{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$$
$$\boldsymbol{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$$

Factoring Quadratic Polynomials aka. 2nd degree polynomials

Ex. $x^2 + 2x - 15$

$$(x \pm \_\_)(x \pm \_\_)$$

Find 2 numbers that multiplied give you 15 and add to 2

Verify your answer by expanding your factors

$$(x + 5)(x - 3)$$

Ex. $\boxed{4x^2} + 10x - 6$

When the coefficient of the x^2 term has more than one pair of positive factors, it means the initial form must be one of the following possibilities:

$$(4x \pm \_\_)(x \pm \_\_)$$
$$(2x \pm \_\_)(2x \pm \_\_)$$

To fill in the blanks find all the factors of -6

(-1)(6)   (1)(-6)   (-2)(3)   (2)(-3)

With some trial and error we can find the correct factoring of this polynomial.

$$(2x - 1)(2x + 6)$$

In this case, we can actually do one more step and factor a 2 out of the second term

$$2(2x - 1)(x + 3)$$

This is important because we could also have factored this as

$$(4x - 2)(x + 3)$$

They look like completely different answers; however, now we can factor a 2 out of the first term which gives us

$$2(2x - 1)(x + 3)$$

Zero of a Function

A value of x which makes a function f(x) equal 0

Ex. 3 is a zero of $f(x) = x^2 - 4x + 3$

This is because $f(3) = 3^2 - 4(3) + 3 = 0$

Finding the zeros

Key formula

$$f(x) = 0$$

Where:

$f(x)$ = The function being evaluated

x = The input value

★ Any value of x satisfying this equation is called a zero of f.
A zero may be real or complex

Step by step:

Step 1: Set the function equal to zero

Step 2: Solve for x

If a quadratic expression ($f(x) = x^2 - 5x + 6$),
then factor and set each factor equal to zero

Step 3: Verify

Multiplicity

How many times a particular number is a zero for a given polynomial

Ex. In the polynomial function:

$$f(x) = (x-3)^4(x-5)(x-8)^2$$

the zero 3 has a multiplicity 4, 5 has a multiplicity 1, and 8 has a multiplicity 2.

Although this polynomial has only 3 zeros, we say that it has <u>7 zeros counting multiplicity</u>.

Key Formula

$$f(x) = a(x-r_1)^{m_1}(x-r_2)^{m_2} \ldots (x-r_n)^{m_n}$$

Where:

$r_1, r_2, \ldots, r_n =$ The distinct zeros (roots) of the polynomial

$m_1, m_2, \ldots, m_n =$ The multiplicity of each corresponding zero

$a =$ The leading coefficient (a nonzero constant)

<u>Step-by-step:</u>

Step 1: Factor out the greatest common factor

Step 2: Factor completely

Step 3: Identify each zero and its multiplicity

Step 4: Check that sum of the multiplicities add to the degree of the polynomial

Inequalities and Absolute Value

If $a < b$ and $b < c$, then $a < c$

If $a < b$, then $a + c < b + c$

If $a < b$ and $c > 0$, then $ca < cb$

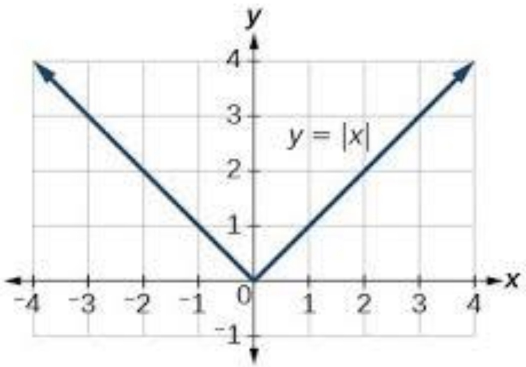
If $a < b$ and $c < 0$, then $ca > cb$

If $a > 0$, then

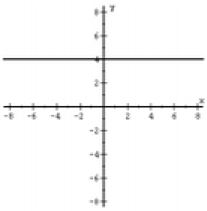
$$|x| = a \text{ means } x = a \text{ or } x = -a$$

$$|x| < a \text{ means } -a < x < a$$

$$|x| > a \text{ means } x > a \text{ or } x < -a$$

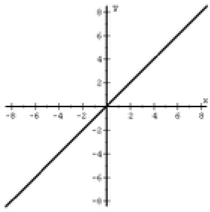


Essential Functions



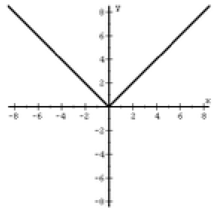
$f(x) = a$

Constant



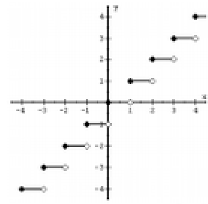
$f(x) = x$

Linear



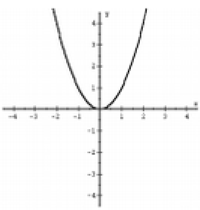
$f(x) = |x|$

Absolute Value



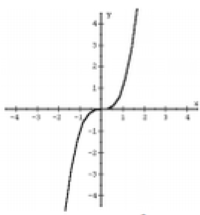
$f(x) = \text{int}(x) = [x]$

Greatest Integer



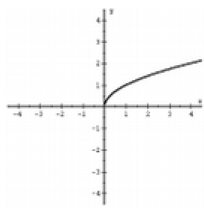
$f(x) = x^2$

Quadratic



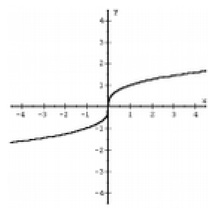
$f(x) = x^3$

Cubic



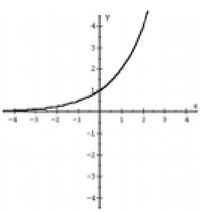
$f(x) = \sqrt{x}$

Square Root



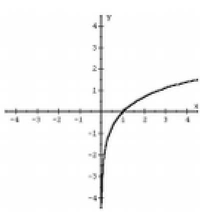
$f(x) = \sqrt[3]{x}$

Cube Root



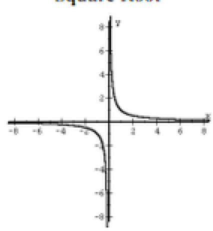
$f(x) = a^x$

Exponential



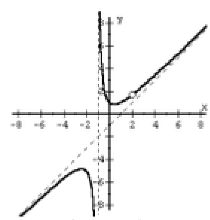
$f(x) = \log_a x$

Logarithmic



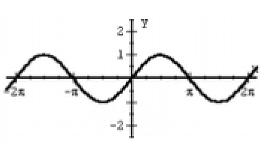
$f(x) = \frac{1}{x}$

Reciprocal

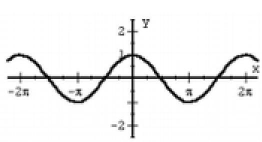


$f(x) = \frac{(x^2+1)(x-2)}{(x+1)(x-2)}$

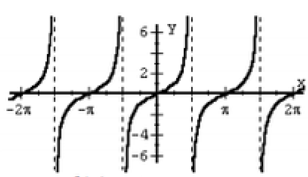
Rational



$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$

Trigonometric Functions

Dividing Polynomials

Long division

Numerator $\frac{x^2 + 5x + 6}{x - 1}$ Denominator $\rightarrow$ $x - 1 \overline{) x^2 + 5x + 6}$

1. Divide the first term of the numerator by the first term of the denominator, and put that in the answer (above the line)
2. Multiply the denominator by that answer, and put that below the numerator
3. Subtract to create a new polynomial

REPEAT!!!

Ex.

$$
\begin{array}{r}
x + 6 \\
x - 1 \overline{) x^2 + 5x + 6} \\
-(x + x) \\
\hline
6x + 6 \\
-(6x - 6) \\
\hline
12
\end{array}
$$

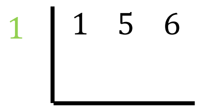
12 ← Remainder

$$\frac{\boldsymbol{x^2 + 5x + 6}}{\boldsymbol{x - 1}} = \boldsymbol{x + 6 + \frac{12}{x - 1}}$$

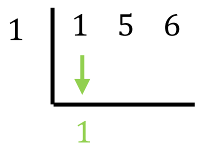
★ Make sure both polynomials have the "higher order" terms first (those with the largest exponents)

Synthetic Division

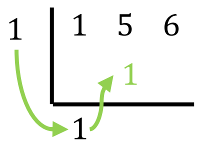
1. Write ONLY the coefficients of the numerator inside an upside-down division-type symbol



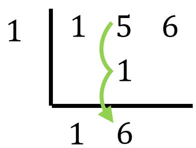
2. Find the "test-zero" by setting the denominator = to 0 & solving ($x - 1 = 0$), place the answer to the left of the #s



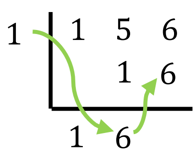
3. Take the first number on the inside, and carry it down, unchanged, below the division symbol



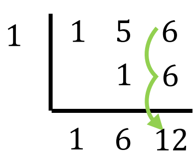
4. Multiply this carry-down value by the test-zero on the left, and carry the result up into the next column inside



5. Add down the column



6. Repeat step 4 with the new carry-down value



7. Add down the column again
8. The last carry-down value is the remainder

Note: Step 2 only works for when the denominator is a binomial

Logarithms

A logarithm answers the question:
How many 2s multiply together to make 8?

$$\underbrace{2 \times 2 \times 2}_{3} = 8$$

Remind you of anything??

$$2^3 = 8$$

We write it like this:

we want to get → $\log_2(8) = 3$ ← logarithm

base

Change of base theorem where "b" is a new base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Common logarithms ⇒ Base 10

- When a logarithm is written without a base, it usually means the base is 10
    - On a calculator it's the "log" button

Natural logarithms ⇒ Base "e"

- Another base that is often used is e (Euler's #) which is about 2.71828…
    - On a calculator it's the "ln" button

Trigonometric functions

The 6 trigonometric functions of θ are defined as follows:

Sine (sin)

$$sin\theta = \frac{y}{r}$$

Cosecant (csc)

$$csc\theta = \frac{r}{y}, y \neq 0$$

Cosine (cos)

$$cos\theta = \frac{x}{r}$$

Secant (sec)

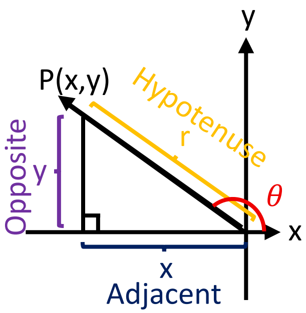
$$sec\theta = \frac{r}{x}, x \neq 0$$

Tangent (tan)

$$tan\theta = \frac{y}{x}, x \neq 0$$

Cotangent (cot)

$$cot\theta = \frac{x}{y}, y \neq 0$$



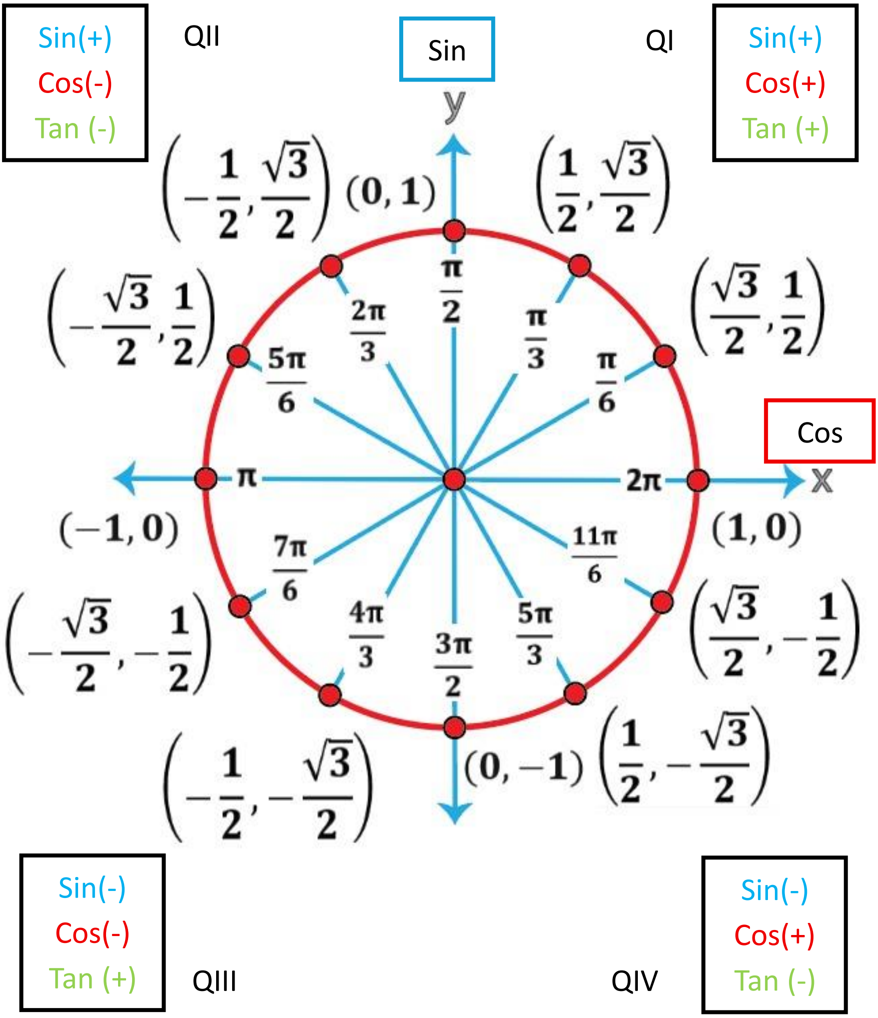
Even-odd Properties

Sin, csc, tan, and cot are odd functions; cos, and sec are even functions.

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

The Unit Circle

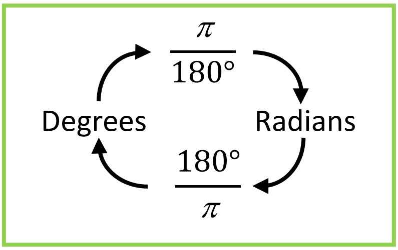


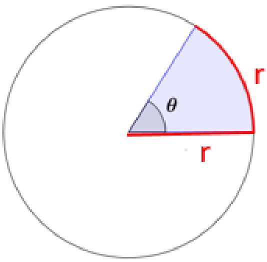
★ The unit circle is a circle of radius 1 centered at (0,0)

Radian Measure

A radian is the measure of a central angle θ whose arc length is the same as the radius of the circle

aka. the unit circle





Arc length = radius

$\theta = 1$ radian

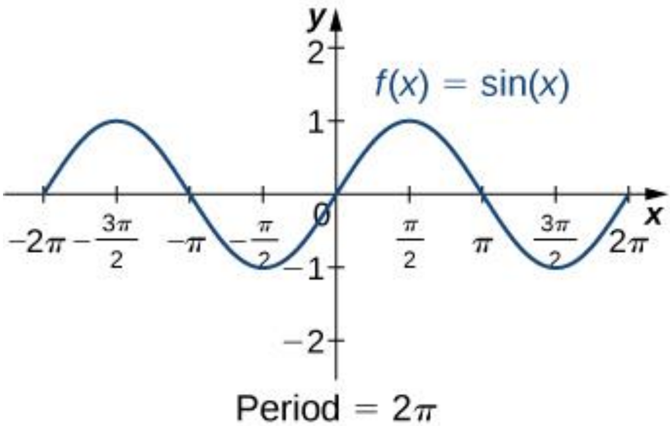
★ Radians $\Rightarrow \pi$

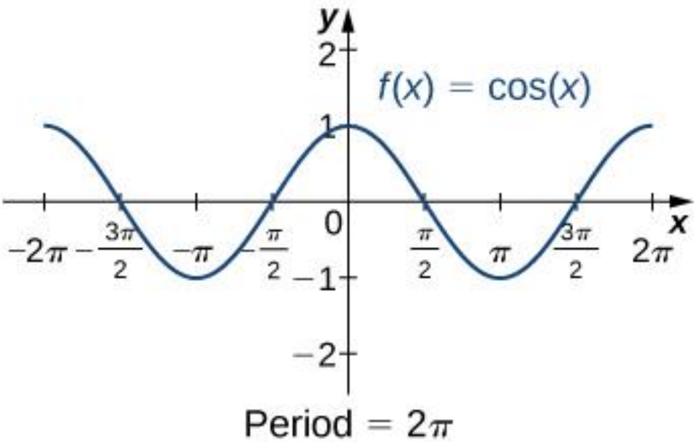
When using the unit circle the trigonometric functions are defined as follows

$$\sin t = y \qquad \cos t = x \qquad \tan t = \frac{y}{x}$$

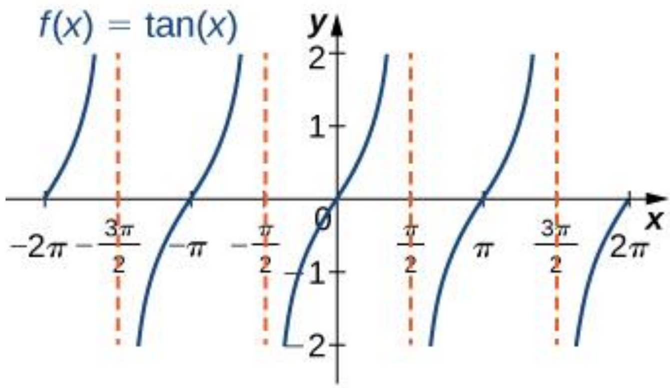
$$\csc t = \frac{1}{y} \qquad \sec t = \frac{1}{x} \qquad \csc t = \frac{x}{y}$$

Graphs of Sin & Cos

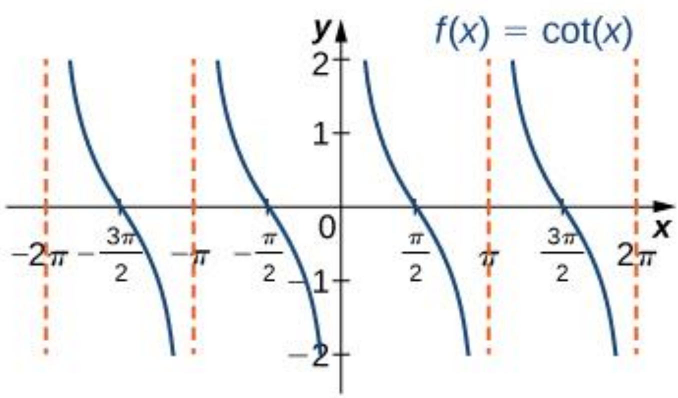




Graphs of Tan & Cot

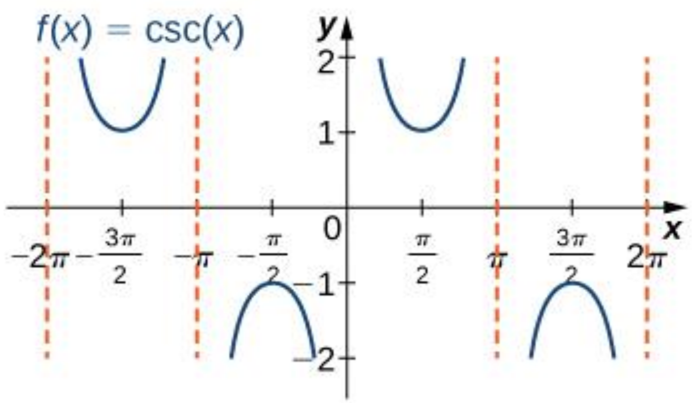


Period = π

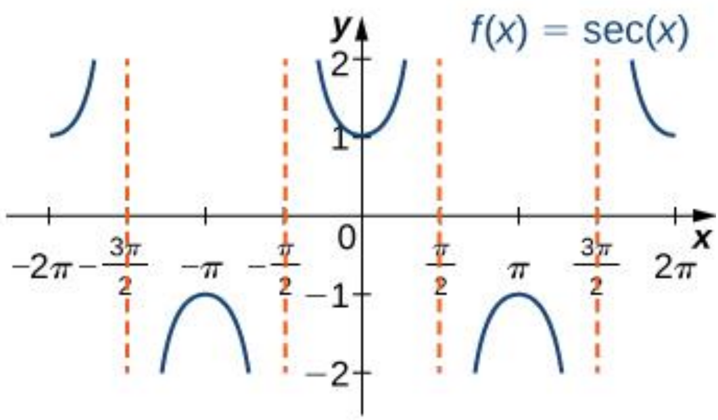


Period = π

Graphs of Csc & Sec



Period = 2π



Period = 2π

Transformation of Trigonometric Graphs

$$y = A\sin(Bx \pm C) + D$$

(+) move to the left
(-) move to the right

$|A|$ = amplitude

$\frac{C}{B}$ = horizontal/phase shift

- If $y = A\sin[B(x \pm C)] + D$,

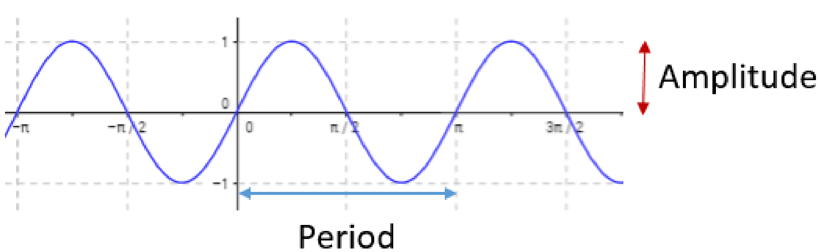
  horizontal shift = C

$$period = \frac{2\pi}{B}$$

D = vertical shift

The same applies for the Cosine function

★ For Tangent, the period is $\frac{\pi}{B}$



Fundamental identities

Show how the trigonometric functions are related to each other

Pythagorean identities

$$sin^2\theta + cos^2\theta = 1$$

$$1 + tan^2\theta = sec^2\theta$$

$$cot^2\theta + 1 = csc^2\theta$$

Quotient identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal identities

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

<u>Using the identities</u>

Find cosθ,

given that secθ = 5/3

$$cos\theta = \frac{1}{sec\theta} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$$

Verify

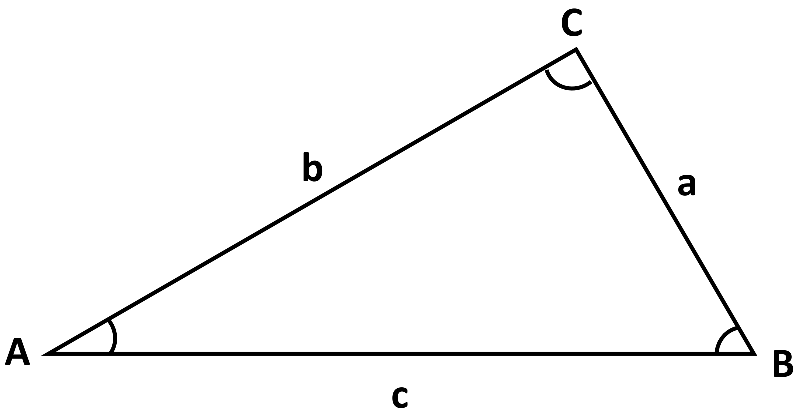
$$tan\theta cos\theta = sin\theta$$

$$= \left(\frac{sin\theta}{cos\theta}\right) cos\theta$$

$$= \frac{sin\theta}{\cancel{cos\theta}} \cdot \frac{\cancel{cos\theta}}{1}$$

$$= sin\theta$$

Law of Sines

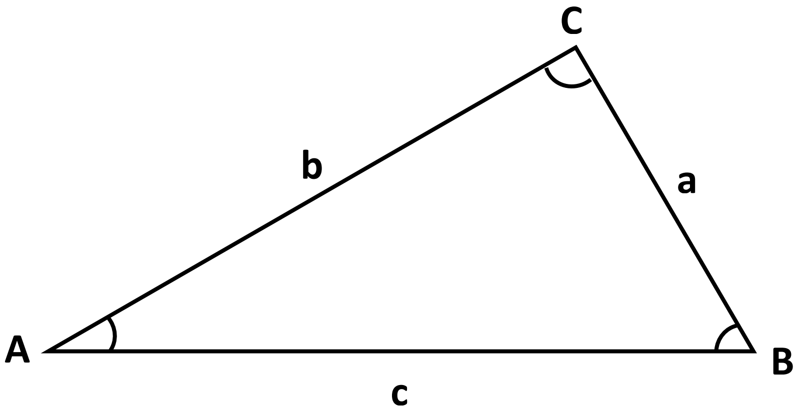


$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

OR

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines



$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

The Binomial Theorem

Go to this value → n

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Start at this value

What to sum

Let's try with n = 3

$(a+b)^3$

$$= \sum_{k=0}^{3} \binom{3}{k} a^{3-k} b^k$$

$$\binom{\boldsymbol{n}}{\boldsymbol{k}} = \frac{\boldsymbol{n}!}{\boldsymbol{k}!\,(\boldsymbol{n}-\boldsymbol{k})!}$$

The "!" means factorial

Ex.

4! = 4 x 3 x 2 x 1 = 24

$$= \binom{3}{0} a^{3-0} b^0 + \binom{3}{1} a^{3-1} b^1 + \binom{3}{2} a^{3-2} b^2 + \binom{3}{3} a^{3-3} b^3$$

$$= \left(\frac{3!}{0!\,(3-0)!}\right) a^{3-0} b^0 + \left(\frac{3!}{1!\,(3-1)!}\right) a^{3-1} b^1 + \left(\frac{3!}{2!\,(3-2)!}\right) a^{3-2} b^2 + \left(\frac{3!}{3!\,(3-3)!}\right) a^{3-3} b^3$$

$$= 1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3 \cdot a^1 b^2 + 1 \cdot a^0 b^3$$

$$= a^3 + 3a^2 b + 3ab^2 + 1b^3$$

By definition 0! = 1

Exponents & Radicals

$$x^m x^n = x^{m+n}$$

$$(x^m)^n = x^{mn}$$

$$(xy)^n = x^n y^n$$

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$

$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$x^{-n} = \frac{1}{x^n}$$

$$(\frac{x}{y})^n = \frac{x^n}{y^n}$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[m]{\sqrt[n]{x}} = \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$$