

# Math



# Must

# Knows!



# Math in Real Time

## Restaurant Receipt

- You're at a restaurant with 5 people. You could've sworn you got the special for \$5 but they overcharged you. How much was the **difference** so you can correct the error?
- After reviewing the bill and splitting the check with your friends, how will each of you split the tip for the waiter?

## Managing your money

- If you want to go to the mall but need mall money, is it better to pay off your loan now or overtime with interest?
- With gas prices changing, what do you need to cut from your average spending to adjust?
- Is a lower interest rate for a longer time better than a higher interest rate for a shorter time?

## Measurements

- Congrats! You won a brand-new PC set up but have a limited amount of space in your room. How will you measure the room and move things around to fit the new PC?

# Whole Numbers

## Place value of digits in numbers



EX: 4,582

- The 4 is in the thousands place so it represents 4,000
- The 5 is in the hundreds place so represents 500.
- The 8 is in tenths place so it represents 80.
- The 2 is in the ones place so it represents 2.

**What about?**

82,360

125,894

1,903,874

# Rounding

Making a number **simpler** but keeping its value close to the original


★ The result is less accurate, but easier to use

**EX:** Round 367 to the nearest **TEN**

-Look at digit to the right, this will be 7

-Since 7 is **5 or more**, round up

thus: 367 rounds up to **370**.

 If the digit you're checking is **5 or more**, round up. If **less than 5**, round down.

**EX:** Round 5,678 to the nearest **HUNDRED**

-Look at the digit to the right of the hundreds (7)

-This digit is larger than 5, so round up

thus: 5,678 rounds up to **5,700**.

**Practice:** Round 564 to the nearest **TEN**

# Comparing Numbers

It is good to know if one number is the **same as**, **smaller than**, or **bigger than** another number:



= (equal to)

EX:  $2+2=4$



< (less than)

EX:  $3<5$



> (greater than)

EX:  $9>6$

EX: Compare the numbers 245 and 254.

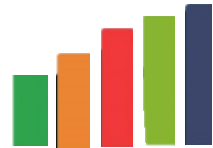
254 is larger than 245, so we write:

**$245 < 254$  OR  $254 > 245$ .**

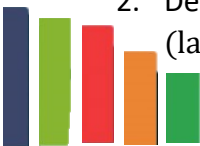
# Ordering Numbers

To **arrange** the numbers in a **specific order**

1. Ascending order aka. from **least** (smallest) to **greatest** (largest)



2. Descending order aka. from **greatest** (largest) to **least** (smallest).



EX: Order the numbers: **100, 56, 78, 89**.

**$56 < 78 < 89 < 100$ .**

# Addition

Combining 2 or more numbers to find total or **sum**.

1. Add the numbers in the **same place value** (right → left)  
★ Stack them!
2. If it adds to more than 9, place the “ones” digit down and add the “tens” digit to the next column
3. Move on to the next column & repeat if needed!

**EX:**  $5 + 3 = 8$  OR  $3 + 5 = 8$

The **sum** of 5 and 3 is 8.

**EX:**  $49 + 35$

$$\begin{array}{r} 49 \\ + 35 \\ \hline 84 \end{array}$$

Diagram illustrating the addition of 49 and 35. The ones column (9 + 5) is highlighted in blue, and the tens column (4 + 3) is highlighted in green. A blue arrow points from the 9 + 5 = 14 to the 4 in the tens column, and a green arrow points from the 14 to the 1 in the tens column. An orange arrow points from the 4 + 3 + 1 = 8 to the 8 in the tens column.

**Practice:**  $686 + 829$

# Subtraction

Taking one number away from another to find the **difference**.

1. Subtract the numbers in the **same place value** (right  $\rightarrow$  left)  
★ Stack them & make sure to put the biggest number on top!
2. If the place value number of the number on top number is less than the one below, borrow a one from the number in the next column

**REMEMBER!** If you borrow a 10 you are **adding 10 to the number on the right** and **subtracting 1 from the number you borrowed from**

EX:  $43 - 7$

$$4 - 1 = 3$$

$$\begin{array}{r} 43 \\ - 7 \\ \hline 36 \end{array}$$

You can't subtract 7 from 3, so you borrow a 10 from 4:  
 $3 + 10 = 13$   
Now you can!  
 $13 - 7 = 6$

EX:  $9 - 4 = 5$

So the **difference** of 9 and 4 is 5.

EX:  $243 - 57$

Do the same thing with the 2

$$\begin{array}{r} 13 \\ 1 \quad 13 \\ 243 \\ - 57 \\ \hline 176 \end{array}$$

You can't subtract 7 from 3, so you borrow a 1 from the 4 to make it a 13  
 $13 - 7 = 6$

**Practice:**  $378 - 298$

# Multiplication

Adding a number to itself a certain number of times  
(**repeated addition**).

EX:  $4 \times 3 = 12$  or  $3 \times 4 = 12$  ← Multiplication tables  
This means:

$$\underbrace{4 + 4 + 4}_{3 \text{ groups of } 4} = 12 \text{ or } \underbrace{3 + 3 + 3 + 3}_{4 \text{ groups of } 3} = 12$$

EX:  $612 \times 4$

$$\begin{array}{r} 612 \\ \times 4 \\ \hline 2,448 \end{array}$$

(3)  $4 \times 6$       (1)  $4 \times 2$   
(2)  $4 \times 1$

★ Multiply the numbers from right → left

EX:  $612 \times 21$

$$\begin{array}{r} 612 \\ \times 21 \\ \hline 1224 \\ + 12240 \\ \hline 12,852 \end{array}$$

(4)  $2 \times 2$       (1)  $1 \times 2$   
(5)  $2 \times 1$       (2)  $1 \times 1$   
(6)  $2 \times 6$       (3)  $1 \times 6$

After the first digit you multiply, shift everything one spot to the left

# Multiplication Table

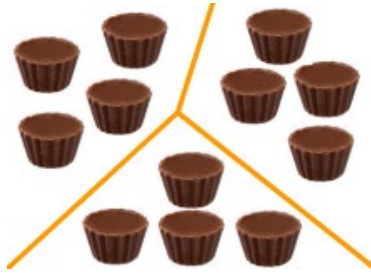
Fill in the multiplication table. Try to work it out and not look at the answers.

x	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

# Division

Splitting into equal parts or groups

Ex. There are 12 chocolates, and 3 friends want to share them. How do they divide the chocolates?



12 divided by 3 is 4. They get 4 each.

We use the  $\div$  symbol, or sometimes the  $/$  symbol to divide

$$12 \div 3 = 4$$

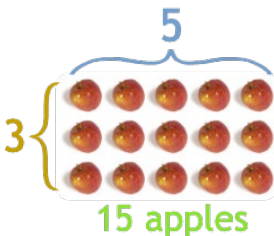
$$12/3 = 4$$

We can also write it this way

$$3 \overline{) 12}$$

Division is the opposite of multiplication

Ex.  $5 \times 3 = 15$ , so  $15/5 = 3$  ...also  $15/3 = 5$



★ Knowing your multiplication tables can help you with division!

### But Sometimes It Doesn't Work Perfectly!!

EX: There are 7 bones to share with 2 pups

$$\begin{array}{r} 3 \\ 2 \overline{)7} \\ \underline{6} \\ 1 \end{array}$$

The number 2 fits into the number 7 exactly 3 times ( $3 \times 2 = 6$ ) so each pup gets 3 bones, but there will be 1 left over.

1 ← This is called the **remainder**.

## Exponents

The **exponent** of a number says **how many times** to multiply it by itself

Base  $\longrightarrow$   $5^3$   $\longleftarrow$  Exponent/power

In this example:  $5^3 = 5 \times 5 \times 5 = 125$

$5^3$  can be called "5 to the third power", "5 to the power of 3" or simply "5 cubed"

In general:

$$a^n = \underbrace{a \times a \times \dots \times a}_n$$

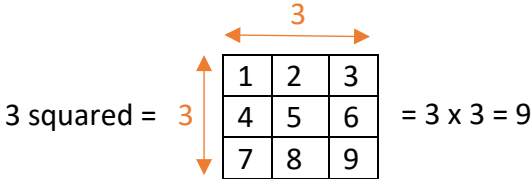
In a calculator

↓  
 $a^n$

# Squares & Square Roots

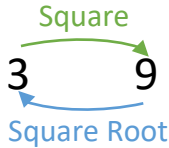
To square a number --> **multiply it by itself**

EX: What is 3 squared?



★ The squares are also on the multiplication table.

A **square root** goes the other way:



3 squared is 9, so a **square root of 9 is 3**

This is the square root symbol aka. the radical →  $\sqrt{9} = 3$

**Perfect squares**

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

$$11^2 = 121$$

$$12^2 = 144$$

What happens when we **square a negative number**?

$$(-5)^2 = (-5)(-5) = 25$$

We get a **positive** number, just the same as squaring a positive number!!

$$(5)^2 = (5)(5) = 25$$

Because of this, the square root of 25 could be **-5 or +5**

# Order of Operations

PEMDAS tells you the correct order to solve operations in a math expression. In other words, what to take care of first. It stands for:

<b>P</b>	<b>( )</b>	<b>Parentheses</b>	
<b>E</b>	<b><math>A^2</math></b>	<b>Exponents</b>	
<b>M</b>	<b><math>\times</math></b>	<b>Multiplication</b>	
or		solve left to right	$2 \times 4$ $(2)(4)$
<b>D</b>	<b><math>\div</math></b>	<b>Division</b>	
			$9 \div 3$ $9/3$
<b>A</b>	<b><math>+</math></b>	<b>Addition</b>	
or		solve left to right	$20 + 5$
<b>S</b>	<b><math>-</math></b>	<b>Subtraction</b>	
			$12 - 8$

SUPERSTARWORKSHEETS

## Examples of PEMDAS

### Solve $6 + 3 \times 4$

1. Is there parenthesis? No
2. Exponents? No
3. Multiplication symbol? Yes  
**Multiply  $3 \times 4$  first**, thus 12.

**Now:  $6 + 12$**

4. Division Symbols? No
5. Addition Symbols? Yes

**Add:  $6 + 12 = 18$ .**

This equation is solved!

### Solve $(4+6) - 2 \div 2$

1. Look for Parenthesis.  
 **$(4+6) = 10$**
2. Look for Exponents. There are none
3. Look for the Multiplication symbols. There are none
4. Look for the Division symbols.

$$-2 \div 2 = -1$$

**Now:  $10 - 1$**  (This can also be written as:  $10 + -1$  and the operation simplifies down to  $10 - 1$ .)

5. Look for any Addition Symbols. There are none
6. Look for Subtraction Symbols.

$$10 - 1 = 9.$$

**PRACTICE: Solve  $(4 - 3) \times 5 + 10 - 1$**

# GCF & LCM

## Greatest Common Factor (GCF):

The largest whole number that divides into two or more numbers

Ex. Find the GCF of 12 and 20

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 20: 1, 2, 4, 5, 10, 20

The GCF of 12 and 20 is 4.

If you are unable to find the GCF, the GCF is 1

## Least Common Multiple (LCM):

The smallest number (not 0) that 2 numbers will divide into evenly

Ex. Find the least common multiple of 3 and 4

Multiples of 3: 0, 3, 6, 9, 12, 15, 18, 21, 24, ...

Multiples of 4: 0, 4, 8, 12, 16, 20, 24, 28, ...

The LMC of 3 and 4 is 12.

# Fractions and Mixed Numbers

Fractions are real numbers that show how many **equal parts** we have from a **whole**

$$\frac{\text{Part}}{\text{Whole}}$$

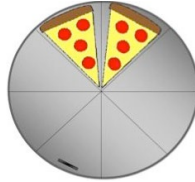
← Numerator  
← Denominator

★ Think of a pizza. The top number says how many slices we **have**. The bottom number says how many equal slices the whole pizza was **cut into**



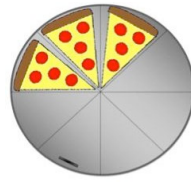
$$\frac{1}{2}$$

One-Half



$$\frac{1}{4}$$

One-Quarter



$$\frac{3}{8}$$

Three-Eighths

## Types of Fractions:

1. **Proper Fractions:** The **part** is **smaller** than the whole.

$$\text{EX: } \frac{3}{5}, \frac{6}{8}, \frac{1}{2}$$

2. **Improper Fractions:** The **part** is **bigger** than, **or equal**, to the whole.

$$\text{EX: } \frac{8}{8}, \frac{10}{3}, \frac{-25}{5}$$

3. **Mixed Numbers:** There is a **whole number** and a **fraction**.

EX:  $2\frac{2}{3}$ ,  $18\frac{1}{8}$ ,  $-9\frac{5}{7}$

## Simplifying

Rewriting a fraction to its simpler form.

1. Is there a **GCF** between the top and bottom numbers?
2. If yes, this fraction can be simplified!  
**Divide** the top and bottom numbers by the GCF.

EX:  $\frac{4}{8}$  ← Both can be divided by 4 =  $\frac{4/4}{8/4} = \frac{1}{2}$

EX:  $\frac{18}{42}$  ← Both are even so they can be divided by 2 =  $\frac{18/2}{42/2} = \frac{9}{21}$  ← Can we keep going? Yes! Both can be divided by 3

=  $\frac{18/3}{42/3} = \frac{3}{7}$  ← Can we keep going? No! These are **prime numbers** and cannot be reduced

### GOLDEN RULE:

Whatever is done to the top, **MUST** be done to the bottom!

## Multiplication

1. Multiply straight across
2. Simplify if needed!

$$\frac{5}{4} \times \frac{2}{3} = \frac{10}{12} \quad \left\{ \begin{array}{l} \text{Can both be} \\ \text{divided by 2} \end{array} \right. = \frac{5}{6}$$

## Division... its like Kentucky Fried Chicken

1. **Keep** the first fraction
2. **Change** the division symbol to a multiplication symbol
3. **Flip** the second fraction
4. Multiply and simplify if needed!

$$\frac{8}{15} \div \frac{1}{4} \quad \text{OR} \quad \frac{8}{15} \frac{K}{\frac{1}{4} \frac{C}{F}} \rightarrow \frac{8}{15} \times \frac{4}{1} = \frac{32}{15}$$

**Improper fraction alert!** Rewrite as a MIXED number

How many times can 15 go into 32? 2 TIMES and the remainder is 2!

So our final answer is:  $2 \frac{2}{15}$

# Adding & Subtracting Fractions

## STEPS:

1. Must have the **same bottom number**, so find the LCM
2. Combine the top numbers **ONLY** (left to right)
3. Simplify (if needed)

EX:  $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$

They have a **common denominator**, so just add the top numbers straight across!

EX:  $\frac{2}{5} + \frac{1}{4} =$

Find the LCM of 5 and 4

Multiples of 5: 0, 5, 10, 15, **20**, 25...

Multiples of 4: 0, 4, 8, 12, 16, **20**, 24...

They both multiply to 20!

Multiply both fractions to get the common denominator

$$\frac{2 \times 4}{5 \times 4} = \frac{8}{20} \quad \frac{1 \times 5}{4 \times 5} = \frac{5}{20}$$

Finally combine the **numerators**

$$\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$

Practice:  $\frac{8}{11} - \frac{7}{22}$

## CONVERT MIXED NUMBERS TO IMPROPER FRACTIONS

1. **Multiply** the whole number to the bottom number
2. **Add** your product to the top number
3. Place your sum at the top and keep the bottom number!

### WHY CONVERT?

Easier to work through problems with!

$$\begin{array}{c} + \rightarrow 3 \\ 2 \frac{\quad}{4} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4} \\ \times \uparrow 4 \end{array}$$

Mixed Number

Improper Fraction

## IMPROPER TO MIXED NUMBERS

$$\frac{14}{3} \rightarrow \begin{array}{r} 4 \\ 3 \overline{)14} \\ \underline{-12} \\ 2 \end{array} \rightarrow 4 \frac{2}{3}$$

Improper Fraction

Mixed Number

### WHY CONVERT?

Tend to be what the final answer is!

# Decimals

← Parts of a whole number

## Decimal Notation and Rounding

Remember the place value of numbers table we went over earlier? Here's an update!!

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths
M	Hth	TTh	Th	H	T	O	•	t	h	th	tth	hth	m
		5	7	8	3	2	.	9	4				

**\$57,832.94**

**SUPERSTAR  
WORKSHEETS**

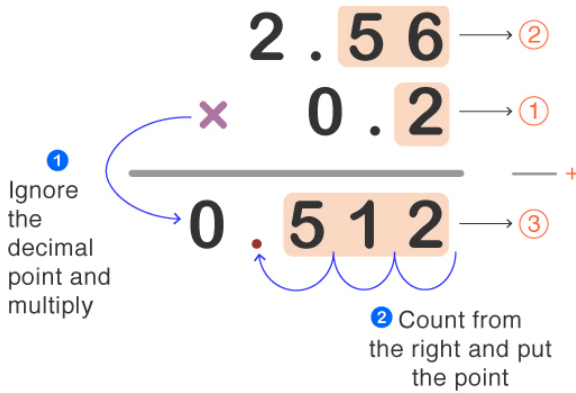
Same thing applies to adding, subtracting and rounding decimals.

★ Just bring down the decimal point.

## Multiplication

### How to Multiply Decimals

1. Multiply normally, ignoring the decimal points
2. **Then** put the decimal point in the answer- it will have as many decimal places as the two original numbers combined



## Division

Let's see how to solve the following examples:

EX:  $24.36 \div 3$

Whole number,  
so no  
problem here

$$\begin{array}{r} 8.12 \\ 3 \overline{)24.36} \\ \underline{-24} \phantom{0} \\ 03 \phantom{0} \\ \underline{-3} \phantom{0} \\ 06 \\ \underline{-6} \\ 0 \end{array}$$

Bring up the decimal  
point & solve as  
normal

EX:  $7.68 \div 0.4$

Make this  
number a  
whole number  
by moving the  
decimal point  
to the **right**

$$\begin{array}{r} 19.2 \\ 0.4 \overline{)7.68} \\ \underline{-4} \phantom{0} \\ 36 \\ \underline{-36} \\ 08 \\ \underline{-8} \\ 0 \end{array}$$

Then move the  
decimal point in  
this number by the  
same number of  
spaces

Bring up the decimal  
point & solve as  
normal

# Percents (%)

When we say “percent” we are really saying “per 100”



Think of your phone battery!!

Percents help us understand numbers in comparison to 100. But what happens when the total amount is not 100??

## Keep this in mind:

100% means ALL

$$100\% \text{ of } 80 \text{ is } \frac{100}{100} \times 80 = 80$$

Ratio:

Compares 2  
numbers by  
division

50% means half

$$50\% \text{ of } 80 \text{ is } \frac{50}{100} \times 80 = 40$$

5% means  $\frac{5}{100}$ ths

$$5\% \text{ of } 80 \text{ is } \frac{5}{100} \times 80 = 4$$

Here's a little calculation trick:

$$x\% \text{ of } y = y\% \text{ of } x$$

EX: **8% of 50** is the same as **50% of 8** = 4

Using **percent ratios** we can solve all kinds of problems:

EX: What is 60% of 90?

# that is 60% of 90  $\rightarrow$  ?  $\times$   $\frac{60\%}{100\%}$   $\div$  Answer = 54

$$\frac{90}{100\%} \times \frac{60\%}{100\%} \div$$

EX: 4 is what percent of 50?

Answer = 8%  $\div$   $\frac{4}{50} \times \frac{?}{100\%}$   $\leftarrow$  Percentage of 4 out of 50

$$\frac{4}{50} \times \frac{?}{100\%} \leftarrow$$

EX: 20 is 75% of what number?

$$\frac{20}{?} \times \frac{75\%}{100\%} \div$$
 Answer =  $26.\bar{6}$

Let's practice:

- Calculate 25 % of 80
- 15% of 200 apples are bad. How many apples are bad?
- If only 10 of the 200 apples are bad, what percent is that?
- Imagine your favorite skateboard is 25% off.  
The old price was \$120. Find the new price.
  - Once you figured out the new price, you realize you still need to calculate how much you have to pay in taxes. If your sales tax is 7.5%, what is your total?
- If after tax you pay \$56.42 for an item that has a sale price of \$52.00, what is the sales tax percentage?

# Decimals, Fractions & Percentages

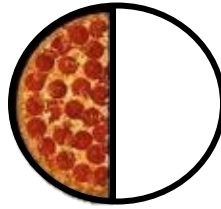
Decimals, Fractions and Percentages are just different ways of showing the same value:

**A half** can be written...

... as a fraction:  $\frac{1}{2}$

... as a decimal: 0.5

... as a percentage: 50%



Here is a table of commonly used values shown in percent, decimal and fraction form:

Percent	Decimal	Fraction
5%	0.05	1/20
20%	0.2	1/5
25%	0.25	1/4
50%	0.5	1/2
75%	0.75	3/4
90%	0.9	9/10
99%	0.99	99/100
<b>100%</b>	<b>1</b>	
125%	1.25	5/4
150%	1.5	3/2

### Decimal to Fraction

$$\frac{\text{decimal}}{\text{place value}}$$

$$0.15 = \frac{15}{100} = \frac{3}{20}$$

### Decimal to Percent

Multiply by 100.  
Decimal moves two places to the right.

$$0.03 = 3\% \quad 1.5 = 150\% \\ 4 = 400\% \quad 0.007 = 0.7\%$$

### Fraction to Percent

1. Change fraction to decimal
2. Change decimal to percent

$$\frac{3}{5} = 5 \overline{)0.6} \quad 3.0 = 60\%$$

### Fraction to Decimal

Divide the top number by the bottom number

$$\frac{1}{4} = 4 \overline{)0.25} = 0.25$$

### Percent to Decimal

Divide by 100.  
Decimal moves two places to the left

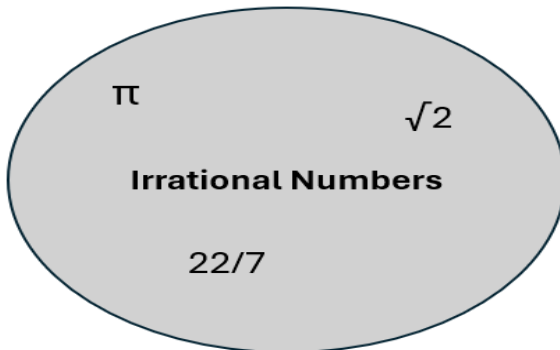
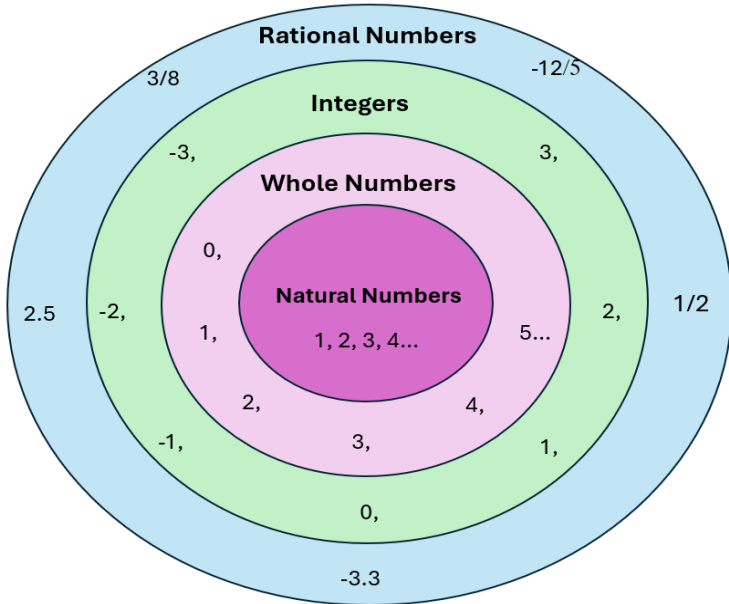
$$17\% = 0.17 \\ 8.5\% = 0.085 \\ 0.25\% = 0.0025 \\ 900\% = 9$$

### Percent to Fraction

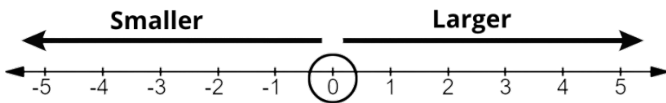
Divide whatever is in front of percent sign by 100

$$35\% = \frac{35}{100} = \frac{7}{20} \\ 225\% = \frac{225}{100} = 2\frac{1}{4}$$

# Real Numbers



# The Real Number Line



Negative numbers (-)    Positive numbers (+)

(The line continues left and right forever.)

## Adding & Subtracting: Negative and Positive Numbers

Sometimes the number line can help you find the right answer

**EX:**  $-4 + 5$

1. **Start from 0.** The **signs (+,-)** tell you where to move on the number line.
2.  $(-4) \rightarrow$  move 4 spaces to the **left**
3.  $(+5) \rightarrow$  move 5 spaces to the **right**
4. This will land you on the answer which in this case is **positive 1**

But what if you have a number that is too big?

**Just remember:**

- Add "like" signs
- Subtract "unlike" signs

★ When subtracting, the bigger number gives its sign to the result

# Multiplying & Dividing: Negative and Positive Numbers

Two major rules:

$(-)(+) = (-)$  Enemy of my friend is my enemy

$(-)(-) = (+)$  Enemy of my enemy is my friend

$(+)(+) = (+)$  Friend of my friend could be my friend

**EX:**  $-3 \times 9 = -18$

**EX:**  $\frac{-8}{-2} = 4$

$|x|$  these lines indicate **absolute value**. The # inside will always be positive!

**EX:**  $|-17| = 17$

# #Common Terms

**Natural Number** — Counting numbers that begin with 1 and increase each time.

**Whole Number** — Counting numbers (natural #'s) including 0.

**Integers** — Positive, and Negative Numbers. Including whole #s and natural #s.

**Rational Numbers** — Numbers that can be written as fractions of two integers.

**Irrational Numbers** — Numbers with decimals that do not terminate or repeat any pattern

**Real Numbers** — **ALL** rational and irrational #s.

**Sum** — Result of addition

**Difference** — Result of subtraction

**Product** — Result of multiplication

**Quotient** — Result of division

**Remainder** — Leftover after division

**Factor** — Divides another number evenly

**Prime Number** — Can only be divided by 1 and itself

**Simplest form** — Fully reduced fraction

**Reciprocal** — flipped fraction

**Variable** — Unknown value (usually x)

**Constant** — Number that does not change

**Expression** — Numbers and operations

**Coefficient** — Number in front of variable → multiplied by variable

**Equation** — Math statement with equal sign

# Multiplication Tables

<b>x</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>1</b>	1	2	3	4	5	6	7	8	9	10
<b>2</b>	2	4	6	8	10	12	14	16	18	20
<b>3</b>	3	6	9	12	15	18	21	24	27	30
<b>4</b>	4	8	12	16	20	24	28	32	36	40
<b>5</b>	5	10	15	20	25	30	35	40	45	50
<b>6</b>	6	12	18	24	30	36	42	48	54	60
<b>7</b>	7	14	21	28	35	42	49	56	63	70
<b>8</b>	8	16	24	32	40	48	56	64	72	80
<b>9</b>	9	18	27	36	45	54	63	72	81	90
<b>10</b>	10	20	30	40	50	60	70	80	90	100

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